Math 115

Summer 2017

Lecture 13

Factoring: GCF → order

$24x^3 - 16x^2 = 8x^2 \cdot 3x - 8x^2 \cdot 2$

$= \underbrace{8x^2}_{\text{GCF}}(3x - 2)$

$75x^4y^2 - 15x^3y^3 = 15 \cdot 5 \cdot x^3 \cdot x \cdot y^2 - 15 \cdot x^3 \cdot y^2 \cdot y$

$= \underbrace{15x^3y^2}_{\text{GCF}}(5x - y)$

Factor out the GCF:

1) $25x^3 - 10x^2 + 5x$

$= 5x(5x^2 - 2x + 1)$

2) $4x^2(2x-3) - 7x(2x-3) + 5(2x-3)$

$= (2x-3)(4x^2 - 7x + 5)$

3) $49x^2y^6 - 14x^6y^2$

$= 7x^2y^2(7y^4 - 2x^4)$

Factor by Grouping (use it when 4 terms or more)

$5x^3 - 3x^2 + 10x - 6$

$= x^2(5x-3) + 2(5x-3)$

$= \boxed{(5x-3)(x^2+2)}$

$7x^3 + 9x^2 - 21x - 27$

$= x^2(7x+9) - 3(7x+9)$

$= \boxed{(7x+9)(x^2-3)}$

Factor

1) $18x^2 - 27x = \boxed{9x(2x - 3)}$

2) $5x^3y^2 - 4x^2y^2 - 10xy^2 + 8y^2$

$= y^2(5x^3 - 4x^2 - 10x + 8)$

$= y^2[x^2(5x - 4) - 2(5x - 4)]$

$= \boxed{y^2(5x-4)(x^2-2)}$

Factoring Trinomials: $ax^2 + bx + c$

$x^2 + 7x + 10 = x^2 + 2x + 5x + 10$

$P = 10$, $S = 7$: $1, 10$; $2, 5$

$= x(x+2) + 5(x+2)$

$= \boxed{(x+2)(x+5)}$

$x^2 - 9x + 14 = x^2 - 7x - 2x + 14$

$P = 14$, $S = -9$: $-1, -14$; $-2, -7$

$= x(x-7) - 2(x-7)$

$= \boxed{(x-7)(x-2)}$

$3x^2 + 10x + 7 = 3x^2 + 3x + 7x + 7$

$P = 21$, $S = 10$: $21 = 1, 21$; $3, 7$

$= 3x(x+1) + 7(x+1)$

$= \boxed{(x+1)(3x+7)}$

$50x^2y + 20xy - 30y$

$= 10y(5x^2 + 2x - 3) = 10y(5x^2 - 3x + 5x - 3)$

$P = -15$, $S = 2$: $-15 = -1, 15$; $-3, 5$

$x(5x-3) + 1(5x-3)$

$= \boxed{10y(5x-3)(x+1)}$

$6x^4 - 20x^3 - 26x^2$

$= 2x^2(3x^2 - 10x - 13)$

$P = -39$, $S = -10$: $-39 = 1, -39$; $3, -13$

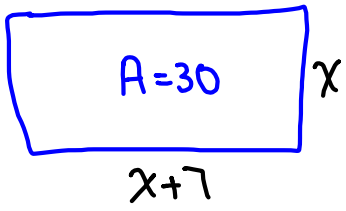
$= 2x^2(3x^2 + 3x - 13x - 13)$

$3x(x+1) \quad -13(x+1)$

$= \boxed{2x^2(x+1)(3x-13)}$

A rectangar garden has an area of $30\ ft^2$.
The length is 7 ft longer than its width.

1) Draw & label such garden.



2) find an expression for its area.

$A = LW$

$= (x+7)x = x^2+7x$

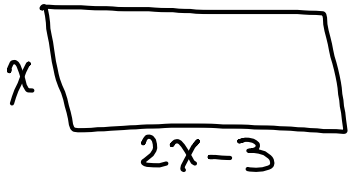
$x^2+7x = 30$

$\boxed{x^2+7x-30=0}$

3) find an eqn in the form of $ax^2+bx+c=0$

The length of a rectangular Pool is 3m shorter than twice its width.

① Draw & label



② find an expression for its area.

$LW = (2x-3)x$

$= 2x^2-3x$

③ write an equation for the area of this rectangle in the form of $ax^2+bx+c=0$ if area of the pool is $54\ m^2$. $2x^2-3x=54$

$\boxed{2x^2-3x-54=0}$

Factor Completely:

① $10x - 15$

$= \boxed{5(2x-3)}$

② $5x^3 + 7x^2 + 5x + 7$

$= x^2(5x+7) + 1(5x+7)$

$= \boxed{(5x+7)(x^2+1)}$

③ $2x^2 - 3x - 54$

$P = -108$, $S = -3$, -108

1, 108	
2, 54	
3, 36	$2x^2 + 9x - 12x - 54$
4, 27	$= x(2x+9) - 6(2x+9)$
6, 18	
9, -12	$= \boxed{(2x+9)(x-6)}$

Special Factoring with Binomials:

$A^2 + B^2 = \text{Prime}$

$A^2 - B^2 = (A+B)(A-B)$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Factor:

1) $x^2 + 9 = x^2 + 3^2 = \boxed{\text{Prime}}$

2) $x^2 - 9 = x^2 - 3^2 = \boxed{(x+3)(x-3)}$

3) $x^2 - 64 = x^2 - 8^2 = \boxed{(x+8)(x-8)}$

4) $x^3 + 64 = x^3 + 4^3$

$= (x+4)(x^2 - 4x + 16)$

5) $x^3 - 64 = x^3 - 4^3$

$= (x-4)(x^2 + 4x + 16)$

6) $x^3 - 49x = x(x^2 - 49) = \boxed{x(x+7)(x-7)}$

7) $2x^4 - 54x = 2x(x^3 - 27)$

$= 2x(x^3 - 3^3)$

$= \boxed{2x(x-3)(x^2 + 3x + 9)}$

8) $3x^5y + 3000x^2y$

$= 3x^2y[x^3 + 1000]$

$= 3x^2y[x^3 + 10^3]$

$= \boxed{3x^2y(x+10)(x^2 - 10x + 100)}$

Factor Completely:

1) $25x^3 - 49x = x(25x^2 - 49)$

$= \boxed{x(5x + 7)(5x - 7)}$

2) $250x^4y - 54xy^4$

$= 2xy(125x^3 - 27y^3)$

$= 2xy[(5x)^3 - (3y)^3] = \boxed{2xy(5x-3y)(25x^2+15xy+9y^2)}$

$x^2(x^2 - 5x - 24) - 100(x^2 - 5x - 24)$

$= (x^2 - 5x - 24)(x^2 - 100)$

$= \boxed{(x + 3)(x - 8)(x + 10)(x - 10)}$

$x^2(x^2 + 6x + 9) + 6x(x^2 + 6x + 9) + 9(x^2 + 6x + 9)$

$= (x^2 + 6x + 9)(x^2 + 6x + 9) = (x+3)(x+3)(x+3)(x+3)$

$= \boxed{(x+3)^4}$

$$2x(8x^3+125)+5(8x^3+125)$$

$$=(8x^3+125)(2x+5)$$

$$(2x)^3+5^3$$

$$=(2x+5)(4x^2-10x+25)(2x+5)$$

$$=\boxed{(2x+5)^2(4x^2-10x+25)}$$

Divide

$$\frac{3x^2-7x+4}{3x-4}$$

$$=\frac{(3x-4)(x-1)}{3x-4}$$

$$=\boxed{x-1}$$

$$3x-4\overline{)\,3x^2-7x+4}$$

$$x-1$$

$$-(3x^2-4x\ \ )$$

$$-3x+4$$

$$-(-3x+4)$$

$$0$$

$\boxed{x-1}$

Divide

$$\frac{8x^3+125}{2x+5}$$

$$= \frac{(2x+5)(4x^2-10x+25)}{2x+5}$$

$$= 4x^2-10x+25$$

$$2x+5 \overline{)\,8x^3+0x^2+0x+125}$$

$4x^2-10x+25$

$-(8x^3+20x^2)$

$-20x^2+0x+125$

$-(-20x^2-50x)$

$50x+125$

$-(50x+125)$

0

$4x^2-10x+25$

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x | $A=30\ ft^2$ | $3x+1$

Find an eqn in the form of $ax^2+bx+c=0$.

$A=30$

$LW=30$

$(3x+1)x=30$

$3x^2+x-30=0$

Factor $3x^2+x-30$ completely.

$3x^2+x-30$

$P=-90$

$S=1$ $\quad -90$

$=3x^2-9x+10x-30$

$=3x(x-3)+10(x-3)$

$(x-3)(3x+10)$

$-1, 90$

$-2, 45$

$-3, 30$

$-5, 18$

$-6, 15$

$-9, 10$

The Sum of Squares of two Cons. integers is 10. Write an eqn in the form of $ax^2+bx+c=0$ using these informations.

x & $x+1$

x^2 & $(x+1)^2$

$$x^2+(x+1)^2=10$$

$$x^2+(x+1)(x+1)=10$$

$$x^2+x^2+2x+1=10$$

$$2x^2+2x+1-10=0$$

$$2x^2+2x-9=0$$

→ factor the LHS.

$P=-18$, $S=2$ → -18

$-1, 18$

$-2, 9$

$-3, 6$

No such exists

The Sum of squares of two Cons. even integers is 20. using these informations, write an eqn in $ax^2+bx+c=0$

x & $x+2$

x^2 & $(x+2)^2$

$$x^2+(x+2)^2=20$$

$$x^2+(x+2)(x+2)=20$$

$$x^2+x^2+4x+4=20$$

$$2x^2+4x-16=0$$

Due tomorrow at 6:00: SG15

Quiz later today

Perfect-Square Trinomials Factoring

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

$x^2 + 16x + 64 = (x + 8)^2$

$x^2 - 24x + 144 = (x - 12)^2$

$9x^2 + 30xy + 25y^2 = (3x + 5y)^2$

$36x^2 - 84xy + 49y^2 = (6x - 7y)^2$

$18x^3 + 60x^2y^2 + 200xy^4$

$= 2x(9x^2 + 30y^2 + 100y^4)$

~~$= 2x(3x \quad 10y^2)^2$~~

$49x^2 - 70x + 25 = (7x - 5)^2$

$625x^4 - 100x^2 + 4$

$= (25x^2 - 2)^2$

Divide $\dfrac{x^4 - 13x^2 + 36}{x^2 + 5x + 6}$

$x^2 + 5x + 6 \overline{)\, x^4 + 0x^3 - 13x^2 + 0x + 36}$

$x^2 - 5x + 6$

$$
\begin{array}{r}
x^2 - 5x + 6 \\
x^2 + 5x + 6 \overline{)\, x^4 + 0x^3 - 13x^2 + 0x + 36} \\
-(x^4 + 5x^3 + 6x^2) \\
\hline
-5x^3 - 19x^2 + 0x + 36 \\
-(-5x^3 - 25x^2 - 30x) \\
\hline
6x^2 + 30x + 36 \\
-(6x^2 + 30x + 36) \\
\hline
0
\end{array}
$$

Looking Ahead:

If $x^2 = K$, $K \geq 0$, then $x = \pm\sqrt{K}$

Square-Root Method

Solve $x^2 = 36$ by S.R.M. $x = \pm\sqrt{36}$

$x = \pm 6$

$\{\pm 6\}$

Solve $(x-1)^2 = 25$ by S.R.M.

$x - 1 = \pm\sqrt{25}$

$x - 1 = \pm 5$

$x = 1 \pm 5$

$x = 6$, $x = -4$ $\{6, -4\}$

Solve $(2x+3)^2 = 49$ by S.R.M.

$1(2x+3) = \pm\sqrt{49}$

$2x + 3 = \pm 7$

$2x = -3 \pm 7$

$x = \dfrac{-3 \pm 7}{2}$

$x = \dfrac{-3+7}{2} = \dfrac{4}{2} = 2$

$x = \dfrac{-3-7}{2} = \dfrac{-10}{2} = -5$

$\{-5, 2\}$

Making a Perfect Square

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

$$x^2 + 10x + 25 = (x+5)^2$$

$$x^2 - 18x + 81 = (x-9)^2$$

$$x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$$

$$x^2 - 11x + \frac{121}{4} = \left(x - \frac{11}{2}\right)^2$$

$$x^2 + \frac{1}{5}x + \frac{1}{100} = \left(x + \frac{1}{10}\right)^2$$

$$\frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$$

$$x^2 - \frac{3}{4}x + \frac{9}{64} = \left(x - \frac{3}{8}\right)^2$$

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

$$x^2 + \frac{2}{7}x + \frac{1}{49} = \left(x + \frac{1}{7}\right)^2$$

$$\frac{1}{2} \cdot \frac{2}{7} = \frac{1}{7}$$

Solving $x^2+bx+c=0$ by completing the square method:

$$x^2-6x+8=0$$

$$x^2-6x+9=-8+9$$

$$(x-3)^2=1$$

Use S.R.M.

$$x-3=\pm\sqrt{1}$$

$$x-3=\pm 1$$

$$x=3\pm 1$$

$$x=4,\ x=2$$

$$\{2,4\}$$

Solve by Completing the sqr method:

$$x^2+10x+21=0$$

Move the Constant term to RHS

$$x^2+10x+25=-21+25$$

Make perfect-square on the LHS

$$(x+5)^2=4$$

Use S.R.M.

$$x+5=\pm\sqrt{4}$$

$$x+5=\pm 2$$

$$x=-5\pm 2$$

$$\boxed{x=-3}\quad\boxed{x=-7}$$

$$\{-7,-3\}$$

Solve by Completing the square:

$x^2 - 3x - 10 = 0$

$x^2 - 3x + \frac{9}{4} = 10 + \frac{9}{4}$

$\frac{1}{2} \cdot 3 = \frac{3}{2}$

$\left(x - \frac{3}{2}\right)^2 = \frac{49}{4}$

by S.R.M.

$x - \frac{3}{2} = \pm\sqrt{\frac{49}{4}}$

$x - \frac{3}{2} = \pm\frac{7}{2}$

$x = \frac{3}{2} \pm \frac{7}{2}$

$x = 5$ $\quad$ $x = -2$

$\{-2, 5\}$

Solving $ax^2 + bx + c = 0$ by

Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

$2x^2 - 3x - 5 = 0$

$a = 2$, $b = -3$, $c = -5$

$b^2 - 4ac = (-3)^2 - 4(2)(-5)$

$= 49$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{49}}{2(2)}$

$= \frac{3 \pm 7}{4}$

$x = \frac{3+7}{4} = \frac{10}{4} = \frac{5}{2}$

$x = \frac{3-7}{4} = \frac{-4}{4} = -1$

$\left\{-1, \frac{5}{2}\right\}$

Factor Comp.:

1) $24x - 9xy$

2) $2x^3 + 3x^2 + 4x + 6$

3) $4x^2 - x - 5$